Chapter 11 - Work and Energy

Introduction

For a layman the term 'work' implies any activity resulting in muscular or mental exertion. In physics, however, the term has a different meaning. It represents a physical quantity.

When a force acts on an object and the object moves in the direction of force, we say that the force has done work on the object.

If you push a book lying on a table you exert force on the book and the book moves in the direction of the force. We say that the force has done work.

If you push a wall, the act will definitely tire you, but the wall does not move. Scientifically, no work is done.

Work and Measurement of Work

Work is said to be done when a force acts on an object and the point of application of the force moves in the direction of force.

Conditions to be satisfied for work to be done:

- Some force must act on the object
- The point of application of force must move in the direction of force

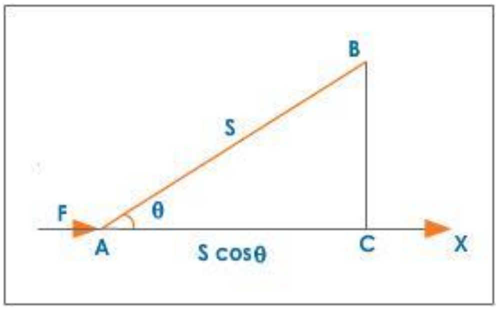
The product of the force and the distance moved measures work done.

$W = F \times S$

Where W is the work done, F is the force applied and S is the distance covered by the moving object. Work done is a scalar quantity.

Work done when the Force is not Along the Direction of Motion

Let a constant force F acting on a body produce a displacement S as shown in the figure. Let □ be the angle between the direction of the force and displacement.



Displacement in the direction of the force = Component of S along AX

= AC

But we know that $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$ $\quad \cos\theta = \dfrac{AC}{S}$

$\therefore \quad AC = S\cos\theta$

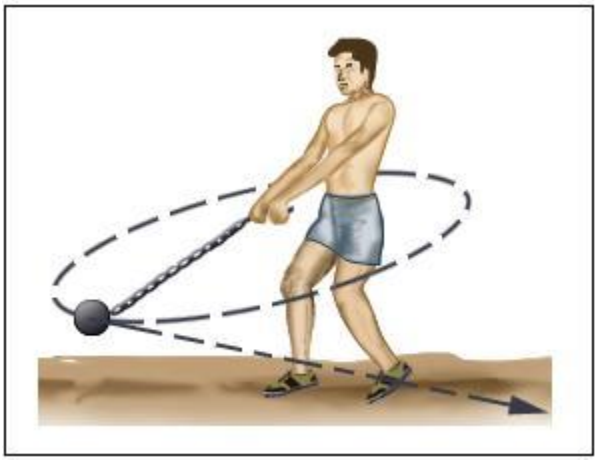
Displacement in the direction of the force = S cosθ
Work done = Force $\times$ displacement in the direction of force

$$W = FS \cos \theta$$

If the displacement S is in the direction of the force F S = 0, cos θ =1
Then, W = FS $\times$ 1

$$W = FS$$

If θ = 90°, cos 90° = 0
Therefore, W = FS $\times$ 0 = 0 i.e, no work is done by the force on the body.
If a stone tied at the end of a string is whirled around in a circle with uniform speed, the centripetal force comes into action. This force is normal to the direction of motion of the stone at each instant. So this force does no work though it is responsible for keeping the stone in circular motion.



SI Unit of Work
W = F $\times$ S
SI unit of F is N and that of S is m [N = newton]
$\therefore$ SI unit of work = N $\times$ m.
1N m is defined as 1 joule.
i.e., 1 joule = 1 N m
$\therefore$ SI unit of work is joule
One joule is the work done when the point of application of a force of one newton moves through a distance of one metre in the direction of force.
1 N m is referred as joule after the British Scientist James Prescott Joule.
The letter 'J' denotes Joule.
Higher units of work are kilojoule and megajoule.
1 kilojoule = 1000J
or 1 kilojoule = 10^3J
1 Megajoule = 1000,000J
or 1 Megajoule = 10^6J

Energy

Anything that is able to do work possesses energy. Energy is the capacity to do work. Energy is measured by the amount of work that a body can do. Therefore, SI unit of energy is also joule.

Different Forms of Energy

There are different forms of energy like for example, mechanical energy, heat energy, electrical energy and chemical energy. In this chapter, we shall study about mechanical energy.
Kinetic energy and potential energy are the two types of mechanical energy.

Kinetic Energy

It is a matter of common experience that a fast moving stone can break a windowpane, falling water can rotate turbines and moving air can rotate windmills and propel sailboats. In all these examples, the moving body possesses energy. Work is done by the body in motion. This type of energy possessed by moving objects, is known as kinetic energy.
“Kinetic energy is defined as the energy possessed by an object by virtue of its motion. Kinetic energy is represented by the letter 'T'. All moving objects possess kinetic energy”.

Expression for Kinetic Energy of a Moving Body

Consider a body of mass 'm' which is initially at rest. When a force 'F' is applied on the body, let it start moving with a velocity 'v' and cover a distance 'S'. The force produces acceleration 'a' in the body.
The force 'F' does work when it moves the body through a distance 'S' and this work done is stored in the body as its kinetic energy.
By definition, $W = F \times S$ ...(1)

$F = ma$ [Newton's second law of motion]

$W = mas$ ...(2)

Also, $v^2 - u^2 = 2aS$ [Newton's third law of motion]

$v^2 - 0 = 2aS$ [Initial velocity u = 0 as the body is initially at rest]

$v^2 = 2aS$

or $a = \frac{v^2}{2S}$

Substituting the value of 'a' in equation (2) we get,

$$W = \frac{mv^2}{2S} S$$

$$W = \frac{mv^2}{2} \quad ...(3)$$

But since work done is stored in the body as its kinetic energy equation (3) can be written as

$$\text{Kinetic energy (T)} = \frac{1}{2} m v^2$$

$$T = \frac{1}{2} m v^2$$

From the above equation we can conclude that the kinetic energy of a body is directly proportional to (1) its mass and (2) the square of its velocity.

Momentum and Kinetic Energy

We know that all moving objects possess momentum. Momentum of the body is defined as the product of its mass and the velocity.

Now let us see how kinetic energy of a body is related to its momentum.

Consider a body of mass 'm' moving with a velocity 'v'. Then, momentum of the body is got by

p = mv

$$\therefore \; v = \frac{p}{m}$$

$$\text{But kinetic energy, } T = \frac{1}{2} m v^2 \qquad ......(1)$$

Substituting the value of v in equation (1) we get,

$$T = \frac{1}{2} m \left(\frac{p}{m}\right)^2$$

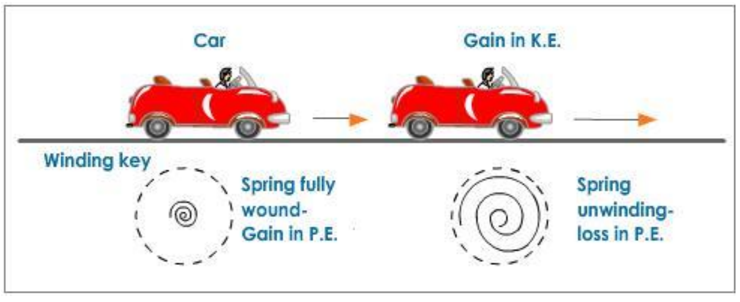
$$= \frac{1}{2} m \frac{p^2}{m^2}$$

$$T = \frac{p^2}{2m}$$

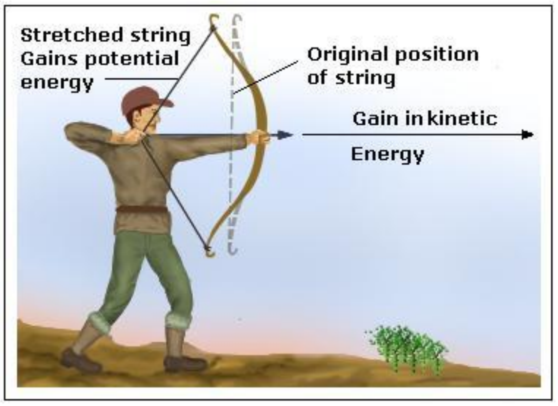
Potential Energy

Study the Following Examples:

- Water stored in a reservoir is capable of rotating turbines kept at a lower level. Water stored in a reservoir possesses energy by virtue of its position.
- A nail hit hard with a hammer gets fixed while if the hammer is just placed on the nail, the nail hardly moves. The hammer, which is raised, possesses energy by virtue of its position.
- A toy car driven by a winding key: When we turn the key the spring gets wound. When we let go, the wheels of the toy car start turning due to the unwinding of the spring, and the car moves if left on the floor. Wound spring possesses energy. The gain in energy is due to the position or condition of the spring.



A Toy Car Driven by a Winding Key



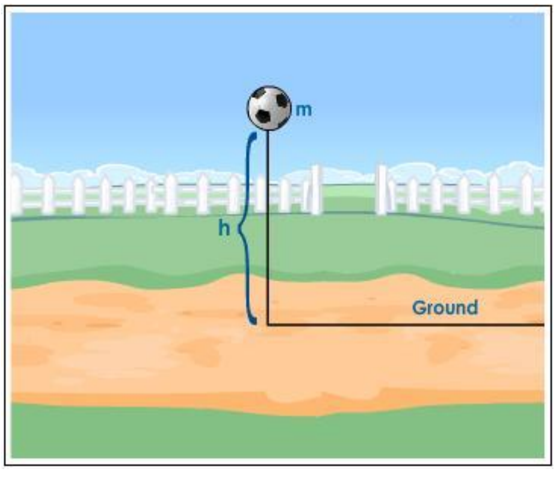
Stretched String Gains Potential Energy

Potential energy of an object can be defined as the energy possessed by the object by virtue of its position or condition.

Expression for Potential Energy

Consider an object of mass 'm', raised through a height 'h' above the earth's surface. The work done against gravity gets stored in the object as its potential energy (gravitational potential energy).

Therefore, potential energy = work done in raising the object through a height 'h'.



Object of Mass 'm', Raised Through a Height 'h'

Potential energy = F × S ...(1)

But F = mg [Newton's second law of motion]

S = h

Substituting for F and S in equation (1), we get

Potential energy = mg × h

∴ Potential energy = mgh

From the above relation it is clear that the potential energy of an object depends on the height from the ground.

Law of Conservation of Energy

Let us see what is actually taking place in the following examples:

- **Steam engine**: The coal burns. Heat due to the combustion of coal converts water into steam. The expansive force exerted by the steam on the piston of the engine moves the locomotive.

Chemical energy is converted to heat energy and heat energy is converted to the expansive force of steam. This transforms to kinetic energy when the locomotive moves.

- **Hydroelectric power plant**: Water stored in a reservoir is made to fall on turbines which are kept at a lower level and which in turn are connected to coils of an a.c. generator.

Potential energy of the water in the reservoir changes to kinetic energy and kinetic energy of the falling water to kinetic energy of the turbines, which in turn changes to electrical energy.

Thus, it is clear that whenever energy in one form disappears, an equivalent amount of energy in another form reappears, so that the total energy remains constant.

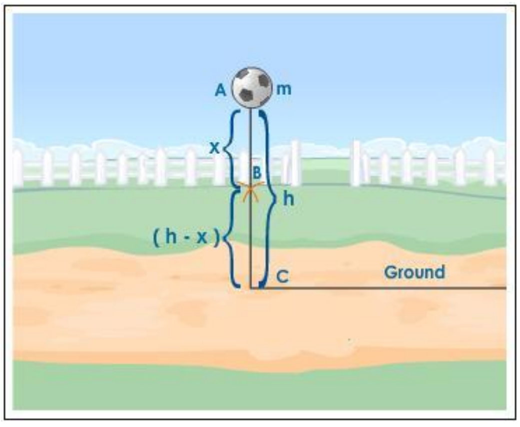
Law of Conservation of Energy

"Law of conservation of energy states that the energy can neither be created nor destroyed but can be transformed from one form to another".

Let us now prove that the above law holds good in the case of a freely falling body.

Let a body of mass 'm' placed at a height 'h' above the ground, start falling down from rest.

In this case we have to show that the total energy (potential energy + kinetic energy) of the body at A, B and C remains constant i.e, potential energy is completely transformed into kinetic energy.



Body of Mass 'm' Placed at a Height 'h'

At A,

Potential energy = mgh

Kinetic energy = $\frac{1}{2}mv^2 = \frac{1}{2} \times m \times 0$

Kinetic energy = 0 [the velocity is zero as the object is initially at rest]

Total energy at A = Potential energy + Kinetic energy

= mgh + 0

Total energy at A = mgh …(1)

At B,

Potential energy = mgh

= mg(h - x) [Height from the ground is (h - x)]

Potential energy = mgh - mgx

Kinetic energy $= \frac{1}{2}mv^2$

The body covers the distance x with a velocity v. We make use of the third equation of motion to obtain velocity of the body.

$v^2 - u^2 = 2aS$

Here, u = 0, a = g and S = x

$v^2 - 0 = 2gx \quad v^2 = 2gx$

Kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}m2gx$

Kinetic energy = mgx

∴ Total energy at B = Potential energy + Kinetic energy

$= mgh - mgx + mgx$

Total energy at B = mgh …(2)

At C,

Potential energy = m x g x 0 (h = 0)

Potential energy = 0

Kinetic energy $= \frac{1}{2}mv^2$

The distance covered by the body is h

$v^2 - u^2 = 2aS$

Here, u = 0, a = g and S = h

$v^2 - 0 = 2gh \quad v^2 = 2gh$

Kinetic energy $= \frac{1}{2}mv^2$

$= \frac{1}{2}m \times 2gh$

Kinetic energy = mgh

∴ Total energy at C = Potential energy + Kinetic energy

= 0 + mgh

Total energy at C = mgh …(3)

It is clear from equations 1, 2 and 3 that the total energy of the body remains constant at every point. Thus, we conclude that law of conservation of energy holds good in the case of a freely falling body.

Power

Imagine two students positioned at track A and track B of 100 m length shifting 10 bricks from one end of the track to the other end. What is the amount of work done by each one of them? The amount of work done is same but the time taken to perform the work varies. In order to find out the fastest among the two we calculate the work done in unit time. That means work done and work done in unit time is two different quantities.

Work done in unit time or rate of doing work is defined as power.

Power is denoted by the letter 'P'.

$P = \frac{W}{t}$ Where 'w' is the work done and 't' is the time taken.

As energy is the capacity to do work, power can also be defined as energy consumed in unit time.

i.e, $P = \frac{E}{t}$ where E is the energy consumed.

SI unit of power

$$P = \frac{w}{t}$$

SI unit of work is joule and time is second. Therefore, SI unit of power is joule/second.

1 joule/second = 1 watt

If one joule of work is done in one second by any agent then its power is said to be one watt.

Kilowatt and megawatt are higher units of power.

1 kilowatt (kW) = 10^3 watts

1 Megawatt (MW) = 10^6 watts

Another unit of power is horse power.

1 horse power = 746 watts

Commercial Unit of Energy

The SI unit joule is too small to express very large quantities of energy. Hence we use a bigger unit called kilowatt hour (kWh) to express energy.

1 kWh is the amount of energy consumed by an electrical gadget in one hour at the rate of 1000 J/s or 1kW.

The energy used in households, industries and commercial establishments are usually expressed in kilowatt hour.

Numerical Relation Between SI and Commercial Unit of Electrical Energy

SI unit of energy is Joule. Commercial unit of energy is kWh.

1 kWh = 1 kW × 1 h = 1000 W × 3600 s

1 watt – second = 1 joule

1 kWh = 3600000 J

$1\text{kWh} = 3.6 \times 10^6 \text{J} = 1\text{unit}.$